



Gostimirovic, M., Kovac, P., Sekulic, M., Savkovic B.

## AN INVERSE HEAT TRANSFER METHOD FOR DETERMINATION OF THE WORKPIECE TEMPERATURE IN GRINDING

**Abstract:** This paper present investigation of the thermal state in grinding process, using inverse heat transfer problems. Based on a temperature measured at any point within a workpiece, this experimental and analytical method allows determination of the maximal temperature on wheel/workpiece interface as well as the complete temperature field in the workpiece surface layer. To solve the inverse heat transfer problems, a numerical method of finite differences in implicit form was chosen.

**Key words:** Grinding process, Temperature field, Inverse problems.

### 1. INTRODUCTION

Unlike other machining technologies, due to lack of a continuous cutting edge, simultaneous cutting of a large number of grinding particles, inconsistent cutting geometry and the transient nature of cutting of a single grinding particle, substantial quantity of thermal energy develops during grinding process. The thermal energy and the problems of its evacuation from the cutting zone account for high contact temperatures which [1]. These increased temperatures instantaneously burst to a maximum, have short duration and exert a pronounced negative effect on wheel surface, workpiece quality and accuracy.

Systematic research in cutting technologies has yielded a number of various analytical and experimental methods for determination of temperatures not only in the narrow and wider cutting zone, but also in the machining system [2]. Due to the rudimentary measuring equipment, first research on temperatures in grinding was mostly theoretical. Later advancements in measuring equipment allowed development of various experimental methods for temperature measurement in grinding, which have been undergoing modification and advancement until today.

Since the main task of grinding is to achieve satisfactory part quality with as large productivity as possible, special attention is focused on the effect that grinding temperatures have on the change of material properties in the workpiece surface layer. If the temperatures thus generated are high enough to cause structural and phase transformations of the workpiece material, the machined surface shall suffer from a number of disadvantages [3]. Should, in addition, dimensional errors appear as well, the overall effect can substantially diminish exploitation features of the finished part.

Efficient machining of parts, free of thermal defects in the surface layer, requires methods for control of thermal phenomena by regulation of

grinding temperatures. For a particular grinding wheel and coolant, temperatures in the cutting zone can be controlled by proper selection of cutting condition parameters [4 and 5].

Although the cutting temperature is an essential parameter in grinding, its utilization for the purpose of control and optimization of grinding is fairly complex. The main obstacle on the road to its utilization for control and optimization purposes lies in the difficult monitoring of cutting temperatures during grinding. Therefore, in order to describe thermal state in grinding, efforts are aimed at improving the existing and development of novel measurement methods, while at the same time focusing on analytical optimization models which can successfully relate to experimental research [6].

As the research so far has shown, non-stationary and non-linear technical processes involving intensive heat transfer, such as grinding, can be successfully solved using novel approach based on inverse heat transfer problems [7, 8 and 9]. Inverse problems allow the closest possible experimental-model approximation of thermal regimes for grinding.

### 2. INVERSE HEAT TRANSFER PROBLEMS

#### 2.1 Thermal process modeling

The process of heat transfer between solid bodies or between a system and its environment, of which heat transfer in grinding is also a part, is mostly considered from the standpoint of mutual relations between input and output process parameters. It is widely accepted that such process can be schematized as in Fig. 1.

The first step in the research of any thermal phenomenon is to model the real process. This means development of a model which is valid over a narrow domain limited by boundary conditions. The model, which describes a segment of the real process, correlates input  $u(t)$  and output  $z(t)$

parameters which define the state of the process at every moment in time  $t$ .

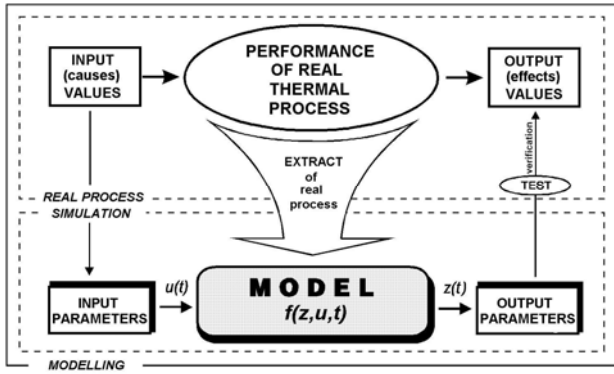


Fig. 1. Diagram of a thermal process

If the input parameters  $u(t)$  are known and output parameters  $z(t)$  define process state in time, then the output parameters are a function of input parameters, i.e.:

$$z = f(u, t) \quad (1)$$

The real thermal process is most often described analytically. The goal is to set up a most adequate analytical model, while, on the other side, keeping its form as simple as possible in order to facilitate solution. Given the right mathematical method, the model thus defined, solves problems quickly and efficiently.

Analytical model of thermal process most often takes form of a system of differential and algebraic equations. Considering the fact that thus modeled task is easily transformed into algorithm and efficiently processed on computer, the differential models are widespread today in the investigation of thermal processes.

## 2.2 General case of inverse method

If for the adopted thermal model there exist unique conditions, then any particular input parameters of the thermal process shall result in that same or any other thermal state defined by the temperature field of the analyzed object. Determination of the input-output relationship is the direct task of heat transfer. Conversely, the inverse method of heat transfer solves the problem of finding input characteristics of the process for the known temperature field.

If for every unknown parameter  $u$  there is a linear, smooth operator  $A$  which allows determination of output parameter  $z$ , the general case of inverse task is formulated by the following equation:

$$Au = z \quad (2)$$

If we represent the unknown input parameter of the thermal state with  $u(t)$ , where  $0 < t < t_K$ , and if  $z(t_K)$  denotes the known output parameter of the process, where  $0 < t_K < t_m$ , then the inverse method

becomes:

$$Au \equiv \int_0^{t_K} u(t)\theta(t_K, t)dt = z(t_K) \quad (3)$$

where  $\theta(t_K, t)$  is the initial characteristic of the process.

In equation (3),  $u(t)$  is the solution, i.e. heat flux or surface temperature of body, while function  $z(t_K)$  represents the temperatures measured outside the body at a point  $K$ .

## 3. INVERSE METHOD OF THE GRINDING PROCESS

The role of mathematical theory behind thermal phenomena in grinding is to adopt the most adequate model of workpiece, grinding wheel and their inter-relationships. It is well known that, due to lack of continuous cutting edge, irregular geometry of grinding particles and their inconsistent distribution on the grinding wheel, it is very difficult to model the grinding process. If the numerous variable parameters were taken into consideration, analytical modeling of the grinding process would become an impossible task. Therefore, some simplifications are necessary where the final solution is verified by experiments. Despite simplification, such analytical and experimental model yields reliable results.

One can assume that the elementary heat source on the grinding particle is the result of friction between the grinding particle, workpiece and chip in the workpiece material shear plane. Summing up all the heat sources, i.e. grinding particles in contact with the workpiece, gives the total heat source for the entire cutting zone,  $q$ . This total heat source, whose strength varies within a narrow range, acts continuously, shifting across the workpiece surface with constant velocity  $v_w$ .

In surface grinding, considering that the cutting depth is many times smaller than the length and width of wheel/workpiece interface, the heat source can be treated as a strip of infinite length and constant heat distribution, Fig. 2. The assumption of constant heat distribution across the interface is valid approximation in case of the heating of thin surface layers of workpiece material.

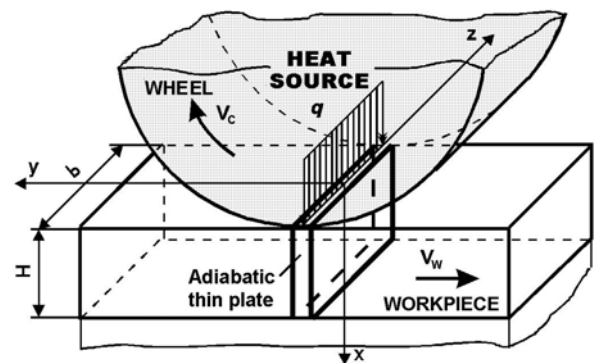


Fig. 2. Model of heat transfer in an elementary workpiece part in surface grinding

If we disregard the dissipation of heat flow in the direction of heat source movement, then the workpiece can be approximated with a semi-infinite plate, or, in other words, be divided up in a series of adiabatic thin plates, Fig. 2. Substitution of the real workpiece with the semi-infinite plate is completely justified, bearing in mind that the heat source in grinding is generated within a small volume of workpiece material while the heat loading of the surface workpiece layer is considered depth-wise.

For such defined thermal model of grinding, the following differential equation general case of a one-dimensional heat transfer:

$$\rho c(\theta) \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left( \lambda(\theta) \frac{\partial \theta}{\partial x} \right) \quad \begin{array}{l} x \in (0, H) \\ t \in (0, t_m] \end{array} \quad (4)$$

where:  $\theta = \theta(x, t)$  - workpiece temperature at point coordinate  $x$  at moment  $t$ ;  $\lambda = \lambda(\theta)$  - heat transfer coefficient;  $\rho c = C(\theta)$  - specific heat capacity ( $\rho$  - material density,  $c$  - specific heat);  $H$  - thickness of the surface layer of workpiece material;  $t_m$  - largest time increment.

Differential equation (4) should be considered in conjunction with the initial temperature distribution and boundary conditions [9].

The initial temperature distribution in the workpiece for the initial moment  $t=0$ :

$$\theta(x, t) \Big|_{t=0} = \varphi(x) \quad x \in [0, H] \quad (5)$$

An additional condition is the fact that at the point  $x=K$  ( $0 < K \leq H$ ), there is a known temperature, measured during a time interval:

$$\theta(x, t) \Big|_{x=K} = \xi(t) \quad t \in [0, t_m] \quad (6)$$

Boundaries of the considered workpiece surface layer are defined by the known temperature at the lower bound of the machined surface:

$$\theta(x, t) \Big|_{x=H} = \bar{\psi}(t) \quad t \in [0, t_m] \quad (7)$$

and unknown temperature at the upper bound of the machined surface:

$$\theta(x, t) \Big|_{x=0} = \psi(t) \quad t \in [0, t_m] \quad (8)$$

The final solution of the inverse problems is the maximal temperature on wheel/workpiece interface  $\psi(t)$ , and the temperature field  $\theta = \theta(x, t)$  throughout entire elementary part of workpiece,  $D = \{(x, t): x \in (0, H), t \in [0, t_m]\}$ .

Due to high complexity, differential equations of the second order which describe the process of heat transfer in grinding are mostly solved using numerical methods. These methods transform exact differential equation into approximate algebraic equations.

To solve the partial differential equation (4) an implicit form of the finite differences method was

chosen [7]. The concept of this method very much resembles the physical process, where the temperature at each observed point is calculated after a time increment as the result of heat exchange with the neighboring points.

Implicit form of finite differences extracted from the mesh area is shown in Fig. 3. Based on the five known temperatures at the neighboring points, the temperature at the next moment in time is calculated.

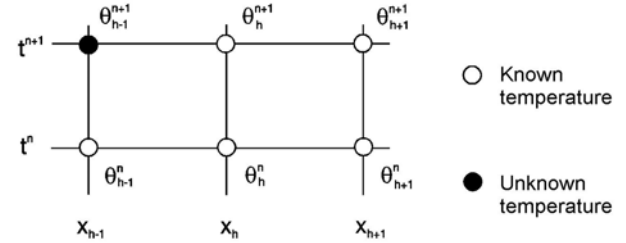


Fig. 3. Numerical method of finite differences in implicit form

System of linear algebraic equations is used to calculate the maximal temperature  $\psi(t)$  on workpiece surface and the temperature field of the workpiece surface layer as follows:

$$[\mathbf{R}] \cdot \{\mathbf{\Theta}\} = \{\mathbf{B}\} \quad (9)$$

Solving the matrix system (9) requires the initial task to be divided into two parts.

Direct method of heat transfer within the area  $D_2 = \{(x, t): x \in [K, H], t \in [0, t_m]\}$ . The solution yields the unknown temperatures  $\theta_h^{n+1}$  ( $K+1, \dots, H$ ).

Now, one can tackle the problems of inverse heat transfer in the area of  $D_1 = \{(x, t): x \in [0, K], t \in [0, t_m]\}$ . Starting from the first unknown temperature  $\theta_{K-1}^{n+1}$ , calculated the unknown temperature  $\theta_h^{n+1}$  ( $h=0, \dots, K-1$ ).

#### 4. THE RESULTS USING INVERSE METHOD

As the proposed system uses experiment and analytical model to control the heat loading of workpiece surface layer in grinding, it requires distribution of temperatures to be determined experimentally at a point within the workpiece.

The experiment work was carried out on a surface creep-feed grinding machine »Majejica« type CF 412 CNC. The workpiece material was high-speed steel (DIN S 2-10-1-8) at 66 HRC hardness. The test was used an aluminum oxide wheel »Winterthur« type 53 A80 F15V PMF. The depth of cut was  $a=0,5$  mm, the workpiece speed was  $v_w=5$  mm/s and the wheel speed was  $v_s=30$  m/s. A water-based coolant (emulsion 6%) was used during the grinding test.

For measurement, processing and control of grinding temperatures used modern information system. The temperature was measured in the workpiece surface layer using a thermocouple built into the workpiece at a specified clearance from the wheel/workpiece interface area. Application of

thermocouple is simple, reliable and cost-efficient, and does not interfere with the real cutting conditions.

In this case of verification, to calculate the workpiece heat loading by inverse heat transfer, the known temperature distribution at depth  $z = 1$  mm was taken for additional boundary condition, Fig. 4.

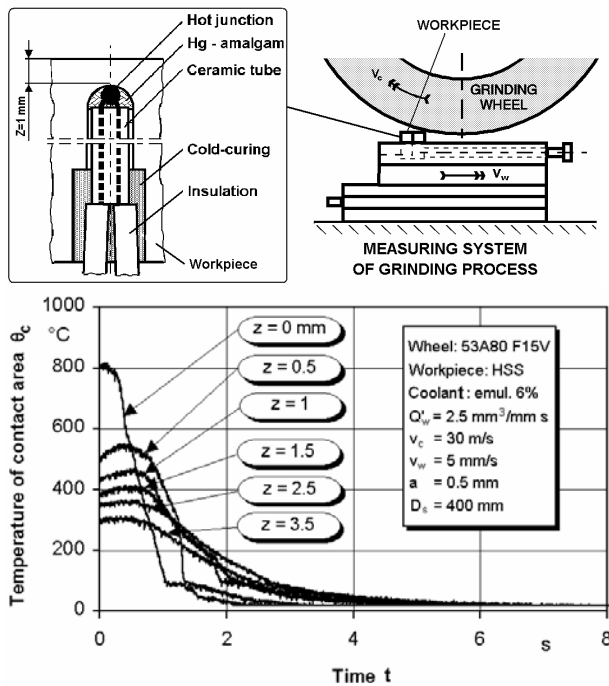


Fig. 4. Experimentally temperature distribution in time within the workpiece surface layer

Based on the previous experimental results, and considering the process boundary conditions and thermal/physical characteristics of grinding, the temperature field in workpiece surface layer was obtained by computation, as well as the maximal temperature on wheel/workpiece interface, Fig 5.

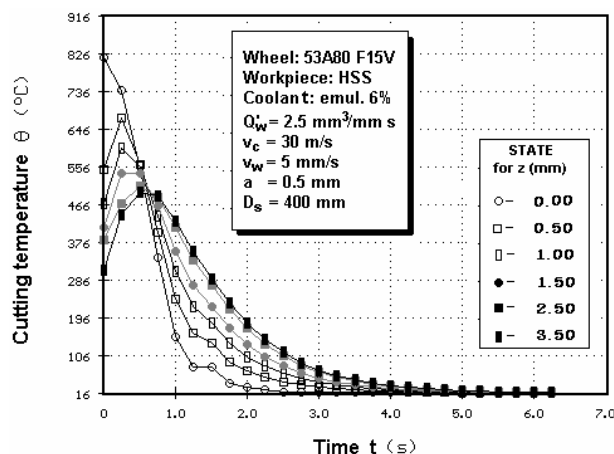


Fig. 5. Computation of temperature change over time in the workpiece surface layer

The computed time and depth-related change of temperature in the interface zone of the workpiece surface layer, shows high degree of conformity with the experimentally obtained results.

## 5. CONCLUSIONS

- Analytical inverse heat transfer method allows approximation of the temperature field in the surface layer of workpiece material with maximal temperature on wheel/workpiece interface;
- The inverse heat transfer method was solved using method of finite differences in implicit form;
- Analytically obtained temperature field in the workpiece surface layer largely agrees with experimental results.

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**Authors:** Assoc. Prof. Dr. Marin Gostimirovic, Prof. Dr. Pavel Kovac, Assis. Prof. Dr. Milenko Sekulic, Borislav Savkovic, dipl. ing., University of Novi Sad, Faculty of Technical Sciences, Institute for Production Engineering, Trg Dositeja Obradovica 6, 21000 Novi Sad, Serbia, Phone.: +381 21 450-366, Fax: +381 21 454-495.

E-mail: [maring@uns.ac.rs](mailto:maring@uns.ac.rs)  
[kovacp@uns.ac.rs](mailto:kovacp@uns.ac.rs)  
[milenkos@uns.ac.rs](mailto:milenkos@uns.ac.rs)  
[savkovic@uns.ac.rs](mailto:savkovic@uns.ac.rs)

*Note:* This paper present a part of researching at the project " *Research and application of high-processing procedure*" Project number TR 14206, financed by Ministry of Science and Technological Development of Serbia.